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Model structure for the economic model utilised within the Healthy Rivers Wai Ora process

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The Technical Leaders Group approves the release of this report to Project Partners and the Collaborative Stakeholder Group for the Healthy Rivers Wai Ora Project.

Signed by:

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**Model structure for the economic model
utilised within the Healthy Rivers Wai Ora
process**

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1. Introduction

The Healthy Rivers Plan for Change: Waiora He Rautaki Whakapaipai (HRWO) Project (www.waikatoregion.govt.nz/healthyrivers) will establish targets and limits for nutrients (N and P), sediment, and *E. coli* in water bodies across the catchments of the Waikato and Waipa Rivers. Different targets and limits regarding the level of these contaminants in waterways within this catchment will have diverse impacts on economic outcomes observed throughout the greater Waikato region. Accordingly, a central contribution of the Technical Leaders Group (TLG) to the HRWO project has been the development and utilisation of an economic model that integrates diverse information such that the size and distribution of abatement costs—across farm, catchment, regional, and national levels—associated with alternative limits and targets was predicted (Doole et al., 2015a, b). The primary objective of this document is to outline the structure of the HRWO economic model utilised at the farm- and catchment-scale, focused on the sources of information and the presentation of the model code.

The report is structured as follows: Section 2 focuses on the model type and the sources of model data, Section 3 describes the model code, and Section 4 concludes. The code for the model itself is presented in Appendix 1 (provided as a separate document due to its length).

2. Model data

2.1 Model type

The model is an optimisation model – that is, it determines the least-cost combination of mitigation measures (land management, land-use changes, and point-source treatments) required to meet the water-quality attribute limits set for each scenario. An automated iterative process is used by the optimisation algorithm to identify how different mitigations could be implemented to minimise the cost associated with achieving a given limit (Bazaraa et al., 2006). The term “optimisation” conveys how the iterative process seeks to *minimise* the cost of a change, and contrasts a simulation approach in which a model user evaluates different scenarios involving pre-defined management activities across the landscape of interest. The particular optimisation model described in this report uses a method known as mathematical programming (Bazaraa et al., 2006).

The model structure is based loosely on that of the Land Allocation and Management (LAM) catchment framework (Doole, 2012, 2015a). The flexibility of this model is demonstrated in its broad utilisation across a number of nonpoint-pollution contexts, both nationally (Doole, 2013; Howard et al., 2013; Holland and Doole, 2014) and internationally (Beverly et al., 2013; Doole et al., 2013). Key benefits associated with the application of the LAM framework are (Doole, 2015):

1. Its flexible structure allows it to be broadly adapted to diverse circumstances, such as the diverse scenarios studied in Doole et al. (2015a, b).
2. The complexity of the model can be altered, depending on the quality and quantity of resources available.
3. The structure of the model allows the use of a broad range of calibration techniques.
4. Models of substantial size can be constructed (Doole, 2010).

The flexibility of the modelling structure has been particularly critical, as the model utilised in this study contains broadly-diverse relationships between land use, land management, contaminant loss, mitigation activity, pollutant attenuation, groundwater flows of nitrogen, and links between loads and concentrations.

The model is large and highly non-linear given that it integrates information from a diverse range of sources, including a broad range of non-linear water quality relationships (e.g. Yalden and Elliott, 2015). The non-linearity of the model potentially challenges the identification of global optima through the use of non-linear programming, given that non-convexity can lead to the existence of multiple local optima (Gill et al., 1981). The chance that the optimisation process converges to a local optima in the application of this model (Doole et al., 2015a, b) is greatly reduced through the use of a global-optimisation routine that starts the nonlinear-programming procedure from a high number of starting points and then refines this set across time to identify the single best (i.e. global) solution. The Multi-Start Nonlinear Programming (MSNLP) algorithm (Ugray et al., 2009) is utilised in the General Algebraic Modelling System (GAMS) (Brooke et al., 2014) for this purpose. A large array of global-optimisation solvers exist (e.g. OQNLP, BARON, LINDO Global), but MSNLP is particularly suited to large problems given its method of operation.

The parameters used to guide the search procedure within MSNLP are set to ensure a very-thorough search is carried out. The default parameters used in MSNLP are set to provide a thorough search for optima among diverse optimisation problems. These are mostly utilised

in their standard form, except the number of maximum solver calls with no improvement is increased from 0 to 50; the number of maximum solver calls is increased from 1,000 to 100,000; a smart-random approach is used to generate trial points through the projection of normal and triangular distributions around variable bounds; and CONOPT is used as the local non-linear optimisation solver (Drud, 1994) because this is more robust than the LSGRG method utilised in the base module.

2.2 Model structure

The area allocated to each enterprise is partitioned by cluster and sub-catchment. A sub-catchment is a part of the drainage basin that drains to a particular monitoring point. A cluster within this is an area for which a certain farm type is deemed representative. For example, a sub-catchment may be partitioned between two clusters for dairy production—each denoting a given type of representative farm (e.g. an extensive dairy farm on pumice soils and an intensive dairy farm on allophanic soils).

Land-use conversion is possible, but mostly only those options for de-intensification are simulated. This is consistent with the need to maintain or improve water quality within the HRWO process (especially given the stringent water-quality goals defined by the *Vision and Strategy for the Waikato River*) and the broad relationship between agriculture and water-quality decline throughout New Zealand (Holland and Doole, 2014). Indeed, extensive experiments with the model indicate that there is very little scope for intensification under the key scenarios of interest to the Collaborative Stakeholder Group (CSG) (Doole et al., 2015a, b). Nevertheless, intensification scenarios are permitted within some discrete scenarios performed in the HRWO process, such as some exploratory situations where the implications of future dairy conversion in the Upper Waikato are investigated.

The area allocated to dairy production and dairy-support activity within each cluster and sub-catchment must equal the amount defined by the input data, minus losses to conversion to dry-stock production or forest. The areas of dairy farming allocated to each representative farm in each sub-catchment are identified in Appendix 1 of Doole (2015).

The area allocated to dry-stock production within each cluster and sub-catchment must equal the amount defined by the input data, plus any additions arising from conversions to dry-stock, minus any conversions to forestry. Conversions from dairy production to dry-stock

only yield additions to intensive dry-stock production. The proportion of land allocated to dry-stock production within each cluster in each sub-catchment is drawn from Romera et al. (2014).

The area allocated to horticultural production within each cluster and sub-catchment must equal the amount defined by the input data, minus any conversions to dry-stock and forestry. The proportion of land allocated to horticulture production within each cluster in each sub-catchment is drawn from Romera et al. (2014).

The area allocated to plantation forest in each sub-catchment must equal the amount defined by the input data, plus any conversions from dairy farming, sheep farming, and horticultural activity.

The model uses historical land-use patterns to constrain land-use changes to realistic levels. This approach was deemed appropriate in this application because it is straightforward to code, much easier to formulate and less prone to error than forcing calibration through the use of arbitrary calibration functions (Doole and Marsh, 2014), draws on regionally-specific data, and is the only land-use calibration method that has a rich theoretical justification (Onal and McCarl, 1991; Chen and Onal, 2012). Historic land-use patterns observed for a sub-catchment provide specific insight into the type of land-use change that can occur there. Indeed, these patterns provide spatial information regarding the implicit aggregate and biophysical factors that guide land-use change within this area. Using this historical information within the catchment model applied here allows the specification of a well-behaved aggregate model, despite lacking data for individual farms (Onal and McCarl, 1991; Chen and Onal, 2012). To use this approach, historic land use for each sub-catchment across 1972–2012 was drawn from the work of Hudson et al. (2015). For each sub-catchment, a weight variable was defined across the set consisting of all of the periods studied by Hudson et al. (2015). The total area allocated to each land use in each sub-catchment in each solution of the model had to equal the product of the weight variable in each year multiplied by the area of this particular land use in this year in that particular sub-catchment, summed across all periods. The (proportional) weight variable could take any value across the closed interval [0, 1], but when summed across all years had to be less than or equal to unity. In this way, the optimisation procedure identified the best weighted-average of historical land-use patterns that attained the environmental limits set out by each scenario at least cost.

The number of mature, rising one-year old, and rising two-year old cows grazed off dairy farms must be balanced with the number of mature, rising one-year old, and rising two-year old cows that can be grazed on dairy-support blocks, dry-stock farms with dairy-support activities, and outside of the catchment. The proportion of cows grazed outside of the catchment is taken from Romera et al. (2014).

The nitrogen model embedded within the economic model replicates the structure of that developed by Semadeni-Davies et al. (2015a). Baseline loads of each diffuse and point source are replicated within the base model. Mitigation activity reduces these loads. A full list of the mitigations that affect nitrogen loss are provided in Doole (2015b) and related documents (e.g. Keenan, 2015).

The sediment model embedded within the economic model contains equations for sediment loads arising from dairy, dry-stock, horticulture, and all other land uses within each sub-catchment. These sediment losses are further partitioned between hillslope and streambank erosion. Erosion levels are determined using the New Zealand Empirical Erosion Model (Dymond et al., 2010; Betts, 2015). The partition between streambank and hillslope erosion is drawn from Hughes (2015). The attenuation in the Waikato River hydro-reservoirs is also derived in Hughes (2015). A full list of the mitigations that affect sediment loss is provided in Doole (2015b).

The phosphorus model embedded within the economic model replicates the structure of that developed by Semadeni-Davies et al. (2015a). Baseline loads of each diffuse and point source are replicated within the base model. Mitigation activity reduces these loads. A full list of the mitigations that affect phosphorus loss are provided in Doole (2015b) and related documents (e.g. Keenan, 2015).

The microbial model embedded within the economic model replicates the structure of that developed by Semadeni-Davies et al. (2015b). Baseline loads of each diffuse and point source are replicated within the base model. Microbial loads are defined for both median and 95th percentile quantities in the model; this is required given that some mitigations (e.g. stream fencing) impact both loads differently, and also because both median and 95th percentile microbial concentrations are of interest to the CSG. Mitigation activity reduces the loads defined in the microbial model. A full list of the mitigations that affect microbial loss are provided in Doole (2015b) and related documents (e.g. Keenan, 2015).

Median and maximum chlorophyll-a concentrations are defined in the model according to the relationships outlined by Yalden and Elliott (2015).

Edge-of-field mitigations defined in the model utilise data provided by Chris Tanner (NIWA). This information is summarised in Doole (2015b). Tanner and Semadeni-Davies (2015) identified the total area that a given edge-of-field mitigation could be applied to within a given sub-catchment. This is defined within the model, with the amount of each land use present in this area computed in each run. The proportion of this total area that the different edge-of-field mitigations are applied to is then computed when the model is solved. Contaminant loss from the land uses present within these areas is then reduced to account for the efficacy determined for each type of edge-of-field mitigation presented in Doole (2015b).

The economic model also includes the water-clarity relationships outlined by Yalden and Elliott (2015).

The CSG has defined scenarios that outline the different limits that the attribute levels computed in the model must obey. Sometimes, it is possible that environmental limits cannot be met. For example, model output highlights that this is particularly relevant to sites where 95th percentile *E. coli* loadings are highest in the catchment (Doole et al., 2015a, b). Normally, such violations will cause infeasibility of a mathematical-programming model, as there is no way that all limits can be met subject to the other relationships within the model remaining satisfied. To prevent such disruption to solution of the model, the limits defined within each scenario are formulated as soft constraints through the use of elastic programming (Gill et al., 2005).

The objective function of the model consists of total profit minus the sum of the total penalty value arising from the use of soft constraints in the constraints that define the water-quality limits. The computation of total profit is based on the multiplication of per-unit profit/cost by the number of units utilised. For example, total dairy profit consists of profit per ha for each mitigation scenario within each cluster within each sub-catchment multiplied by the hectares present within each option. Likewise, the cost of stream fencing consists of cost per kilometre multiplied by the total kilometres of fencing performed. Profit from dairy, dairy-support, dry-stock, horticultural, and forestry activity is defined by the input data discussed in Doole (2015b). (Some of this data is confidential, so is not available in its entirety within this source.) The costs of different mitigation activities are outlined in Doole (2015b). Transition

costs, denoting the change in total profit associated with land-use change, are defined in Matheson (2015). Total profit is scaled to millions of dollars in the model. The sum of infeasible deviations across all sub-catchments for which a limit is defined is multiplied by a high constant (100,000). This penalty term allows the violation of limit constraints, but minimises the degree to which such violations can occur given that it ensures that the penalty term in the objective function dominates total profit.

3. Model structure

This section provides an overview of the model code. Table 1 presents a description of each set of equations considered in the model code. The actual code for the optimisation model applied in this study is presented in Appendix 1 (provided as a separate document). It sets out the model code for Scenario 1 evaluated for the Collaborative Stakeholder Group. (All confidential data has been substituted with a random number in the code presented in Appendix 1.)

Table 1. Description of each group of equations in the model code. (The code for the model itself is presented in Appendix 1.)

Equation number (first)	Equation number (last)	Description
1	1924	Area allocated to each cluster on dairy land
1925	2664	Area allocated to each cluster on dairy support land
2665	2738	Area allocated to the first cluster on dry-stock land
2739	3034	Area allocated to the remaining clusters on dry-stock land
3035	3256	Area allocated to each cluster on horticultural land
3257	3330	Area allocated to forest
3331	3404	Total area of dairy land allocated to forest
3405	3478	Total area of dairy land allocated to sheep and beef
3479	3552	Total area of sheep and beef land allocated to forest
3553	3576	Management of point source options
3577	3650	Convex combination of dairy land
3651	3724	Convex combination of dry-stock land

3725	3798	Convex combination of horticultural land
3799	3872	Convex combination of forestry land
3873	3946	Restrict weights for land allocation
3947	4019	Number of cows in each subcatchment
4020	4092	Total cows present on stand-off pad
4093	4165	Proportion of cows on stand-off pad
4166	4176	Management of cows grazing off
4177	4250	Number of cows on farms on poorly-drained soils for which low-rate effluent application is used
4251	4324	Proportion of farms using low-rate effluent application
4325	4398	Baseline nitrogen load from dairy land
4399	4472	Nitrogen load from 2-pond systems
4473	4546	Nitrogen load from dairy land
4547	4620	Nitrogen load from dairy support land
4621	4694	Nitrogen load from dry-stock land
4695	4768	Nitrogen load from horticultural land
4769	4842	Nitrogen load from forested land
4843	4916	Nitrogen load from miscellaneous land
4917	4990	Nitrogen load from point sources
4991	5064	Nitrogen load from each subcatchment
5065	5138	Pastoral load of nitrogen
5139	5212	Load of nitrogen from forest
5213	5286	Load of nitrogen from urban and miscellaneous sources
5287	5360	Load of nitrogen from point sources
5361	5434	Load of nitrogen from geothermal sources
5435	5508	Load of nitrogen from groundwater
5509	5582	Total load of nitrogen from groundwater after attenuation across network
5583	5656	Total load of nitrogen from subcatchment including attenuation of pastoral loads
5657	5730	Total load of nitrogen after mainsteam attenuation
5731	5804	Total load of nitrogen after attenuation across network
5805	5878	Concentration of total nitrogen at each site based on standard method

5879	5952	Median concentration of nitrate at each site
5953	6026	95th percentile concentration of nitrate at each site
6027	6100	Proportion of mitigation emanating from afforestation of dairy land
6101	6174	Proportion of mitigation emanating from afforestation of dry-stock land
6175	6248	Total streambank sediment load from dairy land
6249	6322	Total streambank sediment load from dry-stock land
6323	6396	Total streambank sediment load from horticultural land
6397	6470	Total streambank sediment load
6471	6544	Total hillslope sediment load from dairy land
6545	6618	Total hillslope sediment load from dry-stock land
6619	6692	Total hillslope sediment load from horticultural land
6693	6766	Total hillslope sediment load
6767	6840	Total erosion in each subcatchment
6841	6914	Total sediment load in each subcatchment after attenuation
6915	6988	Baseline phosphorus load from dairy land
6989	7062	Phosphorus load from 2-pond systems
7063	7136	Phosphorus load from dairy land
7137	7210	Phosphorus load from dairy support land
7211	7284	Phosphorus load from dry-stock land
7285	7358	Phosphorus load from horticultural land
7359	7432	Phosphorus load from forested land
7433	7506	Phosphorus load from miscellaneous land
7507	7580	Phosphorus load from point sources
7581	7654	Phosphorus load from each subcatchment
7655	7728	Pastoral and horticultural load of phosphorus
7729	7802	Load of phosphorus from urban and miscellaneous and forest sources
7803	7876	Load of phosphorus from point sources
7877	7950	Load of phosphorus from sediment
7951	8024	Total load of phosphorus from subcatchment including attenuation of pastoral loads
8025	8098	Total load of phosphorus after mainstem attenuation
8099	8172	Total load of phosphorus after attenuation across network

8173	8238	Concentration of total phosphorus at each site based on standard method
8239	8246	Concentration of total phosphorus at each site based on alternative method
8247	8320	Total point source median load of microbes in each subcatchment
8321	8394	Total point source 95th percentile load of microbes in each subcatchment
8395	8468	Total point source and pond median load of microbes in each subcatchment
8469	8542	Total point source and pond 95th percentile load of microbes in each subcatchment
8543	8616	Diffuse median load of microbes in each subcatchment from dairy
8617	8690	Diffuse median load of microbes in each subcatchment from dry-stock
8691	8764	Diffuse median load of microbes in each subcatchment from other sources
8765	8838	Diffuse median load of microbes in each subcatchment in total
8839	8912	Diffuse 95th percentile load of microbes in each subcatchment from dairy
8913	8986	Diffuse 95th percentile load of microbes in each subcatchment from dry-stock
8987	9060	Diffuse 95th percentile load of microbes in each subcatchment from other sources
9061	9134	Diffuse 95th percentile load of microbes in each subcatchment in total
9135	9208	Total median load of microbes in each subcatchment before linkage
9209	9282	Total 95th percentile load of microbes in each subcatchment before linkage
9283	9356	Total median load of microbes in each subcatchment after linkage
9357	9430	Total 95th percentile load of microbes in each subcatchment after linkage
9431	9504	Median concentration after mitigation
9505	9578	95th percentile concentration after mitigation

9579	9587	TN to TP ratios for sites where chlorophyll-a is measured
9588	9596	Exponent for first logistic weight in chlorophyll-a regressions
9597	9605	Exponent for second logistic weight in chlorophyll-a regressions
9606	9623	Denominator for logistic weights in chlorophyll-a regressions
9624	9641	Logistic functions for chlorophyll-a regressions
9642	9650	Median chlorophyll-a regressions for each site
9651	9659	Median chlorophyll-a regressions for each site
9660	9668	Maximum chlorophyll-a regressions for each site
9669	9742	Area of land in dairy in each subcatchment
9743	9816	Area of land in dry-stock in each subcatchment
9817	10556	Define area on which each edge of field mitigation is used for each farm type
10557	10875	Place bounds on the area on which each edge of field mitigation can be used
10876	11022	Place bounds on the area on which each edge of field mitigation can be used
11023	11614	Define efficacy of edge of field mitigation bundle for each contaminant
11615	11616	Mean value of a hectare of dairy land
11617	11618	Mean value of a hectare of dry-stock land
11619	11623	Total opportunity cost of land for each mitigation
11624		Opportunity cost of land for stream fencing
11625	11629	Total explicit cost for each mitigation
11630	11638	Percentage change in chlorophyll-a from baseline
11639	11712	Percentage change in sediment load from baseline
11713	11786	Phytoplankton attenuation
11787	11860	Sediment attenuation
11861	11934	Yellow substance attenuation
11935	12008	Total beam attenuation coefficient
12009	12063	Percentage change in total beam attenuation
12064	12137	Percentage change in black disc horizontal sighting rang
12138	12211	Black disc horizontal sighting range
12212	12220	Limit median Chlorophyll-a concentration

12221	12229	Limit maximum Chlorophyll-a concentration
12230	12303	Limit median Total Nitrogen concentration
12304	12377	Limit median Total Phosphorus concentration
12378	12451	Limit median nitrate nitrogen concentration
12452	12525	Limit 95th percentile nitrate nitrogen concentration
12526	12599	Limit median microbial concentration
12600	12673	Limit 95th percentile microbial concentration
12674	12747	Set target for black disc measurement
12748		Objective function
12749	12776	Computation of elements within the objective function

4. Conclusions

The Healthy Rivers: Plan for Change/Wai Ora He Rautaki Whakapaipai (HRWO) project will establish targets and limits for nutrients (nitrogen and phosphorus), sediment, and *E. coli* in water bodies across the Waikato and Waipa River catchments. As part of this process, the Collaborative Stakeholder Group have considered the economic implications of a broad set of scenarios for water-quality improvement across these catchments. This report describes the structure of the economic model used to generate the farm- and catchment-level economic implications of the water-quality limits defined within these scenarios. This model represents a key contribution of the Technical Leaders Group (TLG) to the Healthy Rivers/Wai Ora process, given that it integrates diverse information generated from a broad array of work streams initiated and managed by this committee.

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